Chapter 7
Inviscid Flow

Problem 7.1. Water (with constant density \( \rho \)) is to be pumped up a hill against gravity \( g \) through a pipe that tapers from area \( A_1 \) at the low point to the smaller area \( A_2 \) at a point a vertical distance \( L \) higher. Assuming inviscid steady flow, and assuming that the velocity is uniform across the tube, determine the pressure \( p_1 \) at the bottom needed to pump at a volumetric flow rate \( Q \) if the pressure at the top is the atmospheric value \( p_0 \).

Problem 7.2. In ancient Egypt, circular cross-sectioned vessels filled with water sometimes were used as crude clocks. The vessels were shaped in such a way that, as water drained from the bottom, the surface level dropped at a constant rate: \( \frac{dh}{dt} = V_s = \text{constant} \). Assuming that water drains from a small hole of area \( A \) and that the flow of water is inviscid, find an expression for the radius of the vessel \( r(h) \) as a function of the water level \( h \) in terms of the other variables. Determine the volume of water needed so that the clock will operate for \( n \) hours.

Problem 7.3. In cylindrical coordinates \((r, \theta)\), the velocity components for a uniform inviscid flow around a circular cylinder are

\[
v_r = U \left( 1 - \frac{a^2}{r^2} \right) \cos \theta, \quad v_\theta = -U \left( 1 + \frac{a^2}{r^2} \right) \sin \theta.
\]  

(7.1)

Here \( U \) is the constant magnitude of the velocity approaching the cylinder, and \( a \) is the radius of the cylinder.

(a) If compressible and viscous effects are negligible, determine the pressure \( p(r, \theta) \) at any point in the fluid in the absence of any body forces. Take the pressure far from the cylinder to be constant and equal to \( p_0 \).

(b) What is the pressure \( p(a, \theta) \) on the surface of the cylinder?

(c) The force per unit length on the cylinder due to fluid pressure is

\[
F = - \int_{\theta=0}^{2\pi} p n a \, d\theta
\]

(7.2)

where \( n \) is the outward unit normal on the cylinder. Calculate \( F \). Discuss your result.