PHYSICS OF FLUIDS **25**, 070501 (2013)

Report on the IUTAM Symposium on Mobile Particulate Systems: Kinematics, Rheology, and Complex Phenomena, Bangalore, India, 2012

P. R. Nott,[1] R. H. Davis,[2] M. Reeks,[3] D. Saintillan,[4] and S. Sundaresan[5]
[1]*Department of Chemical Engineering, Indian Institute of Science, Bangalore 560 012, India*
[2]*Department of Chemical and Biological Engineering, University of Colorado, Boulder, Colorado 80309-0424, USA*
[3]*School of Mechanical and Systems Engineering, Newcastle University, Newcastle-upon-Tyne NE1 7RU, United Kingdom*
[4]*Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*
[5]*Department of Chemical and Biological Engineering, Princeton University, Princeton, New Jersey 08544, USA*

(Received 19 September 2012; accepted 31 May 2013; published online 18 July 2013)

This report summarizes the presentations and discussions conducted during the symposium, which was held under the aegis of the International Union of Theoretical and Applied Mechanics during 23–27 January 2012 in Bangalore, India. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4812639]

I. PREAMBLE

Particle and particle-fluid flows are ubiquitous in nature and in industrial processes, yet their mechanics are not well understood. Several factors contribute to the lack of understanding, such as the complex nature of particle interactions (e.g., friction in dry systems, hydrodynamic interactions in suspensions), the complexity of the microstructure, and the dependence of the macroscopic behavior on the nature of forcing. Though considerable progress has been made in the recent decades, many open questions remain. When the forces exerted by the fluid on the grains are much smaller than the contact forces, the systems are referred to as dry granular flows. Many instances of particulate flows fall in this category, and historically dry granular flows have been studied in the two extreme regimes of slow (quasistatic) flow and rapid flow, which are discussed at greater length in Sec. III. Only recently have researchers begun to investigate the vast "intermediate" regime that lies between slow and rapid flows. The fluid exerts large enough forces on the particles either when its velocity relative to the particles is large (typically in gas-particle flows such as in fluidized beds and pneumatic conveying), or when its viscosity is sufficiently high (typically in liquid-particle systems, as in slurries, paints, and inks) – these two regimes are conceptually important and have been well studied, but situations intermediate between them are also found in nature and industry. The description of gas-particle flows has been largely phenomenological, wherein reasonable forms are chosen for the constitutive models for gas-phase turbulence, interparticle and particle-gas interactions; nevertheless, these models have proved quite useful in deriving qualitative, and sometimes quantitative, information on problems such as bubble dynamics, mixing and convection currents in fluidized beds. Yet, a clearer understanding of the interaction of suspended particles with large and small turbulent structures in the carrier fluid is of great importance in many industrial and environmental processes. In the opposite regime of liquid-particle flows, a large body of literature over the past five decades has concentrated on the regime where particle and fluid inertia are either precisely zero, or small. Despite its apparent simplicity, a suspension of rigid particles in a Newtonian fluid exhibits several interesting complexities, such as non-Newtonian rheology, spontaneous segregation of particles, etc. Though much progress has been made on understanding these problems through experimental, analytical and computational tools, much still remains to be done to understand suspension rheology and dynamics.

A proposal was made to the International Union of Theoretical and Applied Mechanics (IUTAM) for organizing a symposium to bring together experts in this broad area, to make an assessment of the state of current research and suggest directions for future work. A second and equally important motivation was to bring young scholars and graduate students in contact with the experts in the field, in an atmosphere that is conducive to informal interaction and exchange of ideas. The IUTAM approved the proposal, and nominated a scientific committee.[1] This symposium follows a series of conferences on particle and particle-fluid flows, several held under the aegis of the IUTAM. Funding for the symposium was provided by the IUTAM, the National Science Foundation, USA, the Department of Science and Technology, India, Tata Consultancy Services, Anton Paar GmbH, and Accurion GmbH.

The symposium focused on three distinct but complementary topics:

- Flow of granular materials,
- Dynamics and rheology of fluid-particle suspensions,
- Kinematics and statistics of living suspensions.

Forty four presentations were made in all, which were divided thematically over the five days of the symposium. The first speaker for each theme gave an overview of recent developments in the field and summarized the open questions, before proceeding to deliver his or her presentation. Sections II–IV describe the topics that were discussed, and describe briefly the presentations of the participants (not necessarily in the order of presentation). Some of the presentations appear as papers following this report in this Special Topic section of *Physics of Fluids*. This report and the papers in the Special Topic section constitute the proceedings of the symposium. The complete program and book of abstracts for the symposium has been deposited as supplementary information.[2]

II. STOKESIAN SUSPENSIONS

The first day of the symposium was devoted to the rheology and dynamics of Stokesian suspensions, defined as systems of particles suspended in a Newtonian fluid for which the Reynolds number based on the particle size, $Re_p \equiv \rho v_0 a/\mu$, is much smaller than unity. Here ρ and μ are the density and viscosity, respectively, of the fluid, a is the nominal particle size, and v_0 is a scale for the fluid velocity. For such systems, the inertia of the fluid is not of consequence, and the Stokes equations govern fluid motion. When the fluid is a liquid, the Stokes number $St \equiv Re_p(\rho_p/\rho)$, where ρ_p is the particle density, is also very small, whence particle inertia is also not of consequence. Examples of Stokesian suspensions abound in nature (e.g., silt, blood) and industry (e.g., fermentation broths, paints), and despite many decades of investigation, several open questions remain.

John Hinch opened the session with a review of an earlier conference on a similar topic, *The NATO Advanced Study Institute on Mobile Particulate Systems* held in Cargèse, Corsica in 1994, organized by Elisabeth Guazzelli and Luc Oger. He noted that many of the topics addressed in that meeting continue to remain active today: Stokesian dynamics introduced by John Brady, shear-induced migration described by Andreas Acrivos, instabilities in fluidized beds discussed by John Davidson and Yuri Sergeev, and simulations of granular flows presented by Joe Goddard, Hans Herrmann, Stuart Savage, and others. The presentations at the 1994 meeting are summarized in the volume edited by Guazzelli and Oger, with the opening articles by John Hinch and Robert Davis providing an introduction to suspensions.[3] Several meetings on the same or similar themes have been held in the intervening years.[4,69–72]

Hinch's opening remarks were followed by presentations which fell largely into two groups: one where the phenomena described are due to hydrodynamic interactions between the dispersed particles, and another where inter-particle contact plays a significant role. Of the former group, three long-standing problems on the effects of hydrodynamic interactions on sedimenting particles were addressed. Hinch spoke on the apparent paradox relating to the particle velocity fluctuations, where early theoretical calculations[5] based on an isotropic and homogeneous distribution of particles predicted the velocity variance to rise with the container size, but experiments[6] did not show any dependence on the container size. Expanding on an argument first put forth by Luke,[7] Hinch argued that the velocity fluctuations caused by variations in the particle concentration are damped by small vertical stratification;[8] this argument resolves the paradox, and also explains, on the basis of a correlation length for the fluctuations, why the results of some particle dynamics simulations and experiments agree with the theory while others do not. Another interesting problem in sedimentation is the shedding of particles from a blob of suspension (containing particles denser than the fluid) sedimenting in the pure fluid. This problem was first considered by Nitsche and Batchelor,[9] and has subsequently received considerable attention. Maria Ekiel-Jeżewska presented the results of her recent simulations, wherein the blob was idealized as a collection of $2N$ particles placed in two identical regular N-polygons, one placed over the other, and was shown to become unstable. The last problem in the theme of sedimentation was that of segregation of a suspension of a particulate mixture in a horizontal rotating drum. This phenomenon was first observed in dry granular mixtures by Oyama,[10] and followed up later by several others (see, e.g., Ref. 11). Later, Tirumkudulu *et al.*[12] showed that axial segregation occurs even in Stokesian suspensions of uniformly sized particles, leading to bands of high and low particle concentration. In this symposium, Anugrah Singh presented results on experiments and simulations on radial segregation of a Stokesian suspension of a mixture of particles of two densities; the relatively heavy particles form a core, and the degree of segregation increases with increase in their relative density, very much like in a granular mixture.

The rheology of concentrated Stokesian suspensions was one of the major topics of discussion, wherein advances in the experimental techniques for rheometry, visualization techniques, and particle dynamics simulations, were displayed. Several presentations contended that particle contact may play an important role in the rheological response of suspensions. In the majority of the literature, contact is ignored with the argument that the strong lubrication forces prevent it. While this is indeed true for perfectly smooth particles, small asperities on the surface may lead to contact. Batchelor and Green[13] recognized that "surface irregularities" could alter the interactions of two spheres and thereby the viscosity of a dilute suspension; this idea was pursued later by Wilson and Davis,[14] who determined the viscosity in pure extensional flow by assuming a simple interaction model to account for roughness. In the symposium, several presentations attempted to probe the influence of particle contact. Olivier Pouliquen described experiments on a granular material immersed in a viscous Newtonian fluid, using a rheological cell with porous walls that allow the pressure to be held constant. This is a standard experimental protocol to conduct "drained" tests on a dense granular material saturated with water (where contact is all important), but the use of a fluid about a thousand times more viscous than water allows the possibility of hydrodynamic forces being comparable to the contact forces. Pouliquen was able to elucidate a viscoplastic, frictional-type rheology very close to the jamming limit.[15] The presentations of Elisabeth Guazzelli and Guillaume Ovarlez reported measurements using traditional rheometry showing non-Newtonian behavior of dense suspensions. Guazzelli described experiments to determine normal stresses from free-surface deflections around rotating rods and in a tilted trough,[16] and Ovarlez combined rheometry and local MRI measurements to observe a yield stress and discontinuous shear thickening. Both observed shear-induced migration of particles, the former speculating that the contact stress may be at least partly responsible. From the presentations described above, the role of the contact stress on the rheology is unclear: while both Pouliquen and Ovarlez report a viscoplastic stress, the former looks at volume fractions close to jamming in a shear cell that appears to enforce contact, while the latter uses a conventional Couette cell. Nevertheless, these presentations make the case for a more careful evaluation of the contact stress in dense Stokesian suspensions, which has hitherto been ignored.

An experiment where particle contact is of the essence was described by Mahesh Tirumkudulu, where he studied the strength of a packing of colloidal bed compressed by the capillary pressure induced by the menisci on the top layer of particles; he argued that the presence of defects in the bed determines its ultimate strength. Another aspect of suspension strength was discussed by Anke Lindner, who presented high-speed photographs of the elongation and detachment of a particle-laden fluid extruded from a syringe, showing that detachment is accelerated due to the presence of the particles.

A microstructural model for the rheology of very concentrated suspensions was presented by Jeff Morris, based upon the Smoluchowski equation for the pair distribution function. He considered

Brownian suspensions of spherical particles interacting through soft repulsive potentials, and for the first time simulated volume fractions up to 50% over a wide range of the Péclet number (representing the ratio of convection to diffusion). The rheology obtained from pair interactions show shear thinning and thickening and normal stress differences that are qualitatively in agreement with simulation and experimental observations. Fluids that exhibit normal stress differences exhibit kinematic features that are absent in Newtonian fluids, such as the rod-climbing and die-swell effects. Arun Ramachandran demonstrated the existence of secondary currents due to normal stress differences for suspension flow in a square conduit. As in the classical Ericksen theory for viscoelastic fluids, such currents are absent in the Newtonian limit or for flow in axisymmetric channels. He showed results of dye-tracer experiments in a suspension, along with modeling results based on a constitutive model for particle stress in viscometric flows. Discrepancies between experiment and model predictions suggest the need for a more comprehensive model for non-viscometric flows.

Ending the first day's session with a presentation on a topic a bit outside the mainstream of the meeting, Gary Leal described the importance of coalescence in liquid-phase polymer blends consisting of drops of one polymer dispersed in the continuous phase of a second polymer. His studies show that the presence of a block copolymer, acting as a surfactant, inhibits coalescence and results in a finer blend of suspended drops.

While many of the basic themes (e.g., sedimentation, rheology) and concepts (e.g., hydrodynamic interactions, diffusion) for suspensions considered at this symposium were discussed in previous meetings during the past two decades, they have matured (as have many of the attendees!), with a better understanding over broader ranges of parameters now available due to more powerful computational and experimental tools. With further enhancements in computing power and refinements of theoretical and experimental tools, it is anticipated that a more comprehensive understanding of suspension behavior will be developed in the coming years and applications of practical importance addressed.

III. GRANULAR FLOW

Historically, granular flows have been studied in the two extreme regimes of slow/quasistatic flow and rapid flow. In the former, grain inertia plays no role and momentum is transmitted solely by abiding grain contacts, with friction playing a dominant role in determining the stress; in the latter regime, grain contacts are of short duration, and particle inertia is paramount in the transfer of momentum. The role of particle inertia is quantified by the Savage number[17]

$$Sa = \frac{\rho\, \dot{\gamma}_0^2 d_p^2}{\sigma_0},$$

where ρ is the bulk density, d_p is the grain size, and $\dot{\gamma}_0$ and σ_0 are the characteristic shear rate and stress in the flow. In some recent works,[18,19] the inertia number $\mathcal{I} \equiv \sqrt{Sa}$, has been used as a measure of particle inertia. The regime of slow flow corresponds to $Sa \ll 1$, and rapid flow to $Sa \approx 1$. Both these regimes were discussed in the symposium, and a few tentative attempts at bridging them were also discussed.

A. Slow flow

Theories for slow, or quasi-static, granular flow are largely phenomenological, and have been derived with ideas borrowed from metal plasticity and soil mechanics. An important difference between metals and granular materials is that the deformation of the former is essentially at constant volume, whereas volume change or dilatancy is an important aspect of the latter.[20] Moreover, in metals there is rarely an interest in strains larger than O(1), as there is typically catastrophic failure for large strain. In granular *flows* on the other hand, one is interested in the limit of very large strain, as in fluids. For this reason, the flow rule (which is an important part of the constitutive relation for the stress) for granular materials is posed in terms of the strain rate, rather than the strain or strain increment. There is a large body of work on the development of plasticity theories for granular materials; the reader is referred to Refs. 21–23 for reviews.

The second day of the symposium was mostly devoted to slow granular flow, with the presentations addressing the themes of flow-induced anisotropy, non-locality in inhomogeneous systems, particle migration and segregation, and the development of continuum models. In the lead paper, Joe Goddard outlined a general theoretical framework for rheological models for complex fluids, using the formalism of Edelen.[24] A dissipation potential is constructed as a function of the scalar invariants of all the relevant fluxes, such as the strain rate; relations for the forces (such as the stress) are then derived as derivatives of the potential with respect to the corresponding fluxes. He described how this general formalism can be used for developing constitutive models for granular flows, including those that incorporate flow-induced anisotropy using an evolutionary fabric tensor. He also made a link to Stokesian suspensions, and showed in particular that Edelen's formalism can address the problem of particle migration, which may arise from gradients of particle concentration, deformation rate and fabric.

Coming to the theme of anisotropy that was mentioned earlier, it has been known for long[25] that the stress in a granular material is transmitted along preferential paths, referred to as force chains. In a continuum description, force chains may be incorporated using an anisotropic fabric tensor, which together with the strain rate (and scalar variables such as the packing fraction) determines the stress. Two papers in the symposium addressed the issue of fabric anisotropy. Prabhu Nott reported on experimental studies on the quasi-static shearing of a dense granular column in a Couette cell. As a variant of the well-known Janssen problem, the goal was to investigate the effects of side-walls and depth on the shear rheology, with gravity acting normal to the plane of shear. The experiments showed vertical stress profiles that differ qualitatively from intuitive expectations, and predictions based on isotropic plasticity.[26] He suggested that the anomalous stress profile is due to an anisotropic fabric that arises from a combined influence of gravity and shear. Farhang Radjai then presented a 2D constitutive equation deduced from DEM simulations for frictional non-cohesive disks. The model involves an evolution equation for a fabric tensor that incorporates both anisotropy and particle-contact density. The model predicts stress evolution in elementary loading paths and the existence of a limiting "jammed" state.

Nonlocal theories have gained support for granular materials as they have been shown to capture phenomena such as shear localization, which are frequently observed.[27–29] In these theories, the constitutive relations for the stress (and other forces) involve higher gradients of the velocity; they lead to several interesting effects, including a shear-band thickness that scales with particle size, which has been found in previous studies of this kind, and the inception of flow at stresses lower than the homogeneous yield stress. In work aimed at elucidating the effects of non-locality, Ken Kamrin reported on the use of DEM simulations to develop a meso-scale granular plasticity model for steady plastic flow. A plastic-flow model is proposed that involves the Laplacian of an effective "fluidity," which appears in certain non-local models of emulsion rheology and which is reminiscent of the well-known plasticity models of Aifantis and coworkers.[28] In an experimental work relevant to large-gradient or non-local effects, Tejas Murthy reported on their experimental studies of the motion of sand particles around a flat punch. A previously developed image-analysis technique[30] was employed to study the quasi-planar granular displacements under the punch. Distinct regions in the deformation field, along with initiation and propagation of shear bands, were identified. The severe jumps in the displacement field resemble those observed during plastic indentation of other solids, with discrepancies in the free-surface region near the punch.

Two presentations discussed the structure and stability of granular aggregates formed by different methods. Ishan Sharma made a connection with planetary geophysics by pointing out that several near-Earth asteroids are believed to be granular aggregates held together solely by self-gravity. He modeled them as rigid-perfectly plastic materials obeying the Drucker-Prager yield criterion, and using an appropriate flow rule he investigated their stability. As a standard spectral stability analysis cannot be applied to this problem owing to the non-smooth constitutive laws, he pursued an alternate approach based on an energy criterion to evaluate stability.[31] He then catalogued the stability of several near-Earth asteroids. Thorsten Pöeschel presented the geometrical characteristics of heaps formed by the ballistic deposition of spheres and sphere-agglomerates from a circular dropping zone onto a flat surface, using Visscher-Bosterli DEM simulations.[32] Very large heaps manifested up to two external angles of repose, one internal angle of repose and four distinct packing-fraction regions.

The influence of the size of the dropping zone on these characteristics was discussed. When the heap was cut into fragments of chosen characteristic size and allowed to settle repeatedly under ballistic deposition, loosely packed structures having fractal characteristics and robust statistical properties emerged.[33]

Joshua Dijksman presented an experimental study of microscopic particle dynamics by means of an index-matched visualization technique, which is commonly used to visualize particle motion in media that are otherwise opaque due to multiple scattering. The technique was used to study particle motion in the vicinity of a shear band induced by a split-bottom shear cell. In addition to the inertial number of Jop *et al.*,[19] he suggested that another parameter, a "layer number" is necessary to describe particle migration. It is not clear how this parameter relates to continuum models of particle migration of the type discussed elsewhere in this report.

From the presentations, which reflect the current state of the literature, it appears that a theoretical model for slow granular flows that accurately represents the important experimental observations is still lacking. While it is clear that plasticity theory will form the basis for any model, several enhancements to classical isotropic plasticity are probably necessary. Some papers have explored the incorporation of a fabric tensor in the constitutive relation for the stress – though this theme has been covered in the literature for some time, more work in this direction is needed. The incorporation of a micro- or mesoscopic length scale in the model to capture shear bands and other regions of sharp variation, which are a generic feature in slow flows, is another feature that requires further exploration. One aspect that appears to be reasonably well represented in models is the effect of particle inertia, for which phenomenological models[19] seem to be in general agreement with DEM simulations and experimental studies of simple shearing flows. However, more comprehensive models are necessary to describe the stress in more complex flows, for which a better understanding of the intermediate regime (see below) is necessary.

B. Granular flow in rapid and intermediate flow regimes

The third day of presentations featured granular flow in the inertial and intermediate regimes, with papers addressing problems ranging from regime transition, instabilities, size segregation, and flow-induced microstructure. As already noted, in the regime of rapid flows particles interact by collisions of short duration (compared to the time of flight between collisions), and the transfer of momentum is due to the inertia of the particles. This picture resembles that of molecular gases, and hence theoretical treatments of rapid granular flows have used the machinery of the kinetic theory of gases, with suitable modifications to account for the inelasticity of collisions. These theories have been shown to agree well with particle dynamics simulations and some experiments for low to moderate volume fraction of particles and small inelasticity, but deviate for larger volume fraction and inelasticity. The deviation is usually ascribed to increasing deviation from the assumptions of the Chapman-Enskog theory, namely that the departure from equilibrium is small, and that the velocities of colliding particles are uncorrelated. Recent work has concentrated on accounting for the deviations by either direct solution of the Boltzmann equation (without recourse to the Chapman-Enskog perturbation procedure), or to account for correlations in the velocities of colliding particles, which was reflected in some of the presentations. A lot of interesting experimental work too is being undertaken by researchers in the community, and some of their findings are not explained by existing theories – a measure of this too was observed in the symposium.

Sankaran Sundaresan started the day with a review of the three regimes of granular flows – quasi-static, inertial, and intermediate – observed in experiments and in DEM simulations. He then discussed critical scaling that collapses the DEM results on pressure and shear stress in simple shear flows in the vicinity of the jamming volume fraction and proposed simple models for pressure and stress ratio that bridge the different regimes.[34] A simple, albeit *ad hoc* modification to the kinetic theory model for inertial granular flow was shown to capture the DEM results for dense assemblies. V. Kumaran used the problem of flow down an inclined plane to investigate the rheology of granular materials in the rapid and intermediate flow regimes. He examined the effect of base roughness on the dynamics of granular layers, with a random configuration of fixed particles used to form the base. DEM simulations revealed a discontinuous transition from a disordered to an ordered flow state when the ratio of diameters of base and moving particles was decreased below a critical value. The ordered flowing state, which consisted of hexagonally close packed layers of particles sliding over each other, had a higher volume fraction and a higher velocity than the disordered flow. Bagnold's law was satisfied for both flows, but the Bagnold coefficient (defined as the stress divided by the square of the strain rate) in a disordered flow state was an order of magnitude larger than that in the ordered flow state. Stefan Luding presented another study employing DEM simulations to construct workable continuum constitutive models for both rapid and slow flows of granular media.[35] DEM simulations are first employed to gain insight into the microstructure and collisional/contact dynamics. A coarse-graining method is then used to extract the relevant macroscopic fields and thereby continuum constitutive laws, which take into account co-existence of force chains, shear bands, stress and microstructure anisotropy, etc. The approach was illustrated using a split-bottom ring shear cell and inclined plane avalanche flows as examples.

The three presentations just described attempted to go beyond the classical kinetic theory-based models for rapid granular flows. Hisao Hayakawa found the kinetic theory-based model itself to be quite useful for studying the stability of simple shear flow in the absence of gravity. He first showed that the theory developed by Jenkins and Richman[36] is in good agreement with DEM simulations. Emergence of inhomogeneous flow structure is attributed to the loss of stability of the uniform shear flow. A bifurcation analysis of the time-dependent Ginzburg-Landau equation for the disturbance amplitude, which was deduced from a weakly non-linear analysis of the model, revealed that the nature of the bifurcation shifted from supercritical to subcritical as the particle volume fraction was increased.[37]

Ruiz-Suárez reported interesting experimental results on the penetration dynamics of an intruder into an initially static granular bed composed of very light particles. The motion of the intruder appears to excite the inertia of the light particles, which seems to play a key role. When the mass of the intruder is below a critical value, the intruder fired downward into the granular bed comes to rest after some time; it achieves a steady settling velocity if the mass exceeds this critical value.[38] A simple model was used to explain this result. DEM simulations reproduce the observed effects and provide a means of probing the details of granular flow around the intruder. Natalie Vriend described laboratory experiments and DEM simulations of unconfined shallow granular flows in a low-angle V-shaped inclined plane under the action of gravity. The surface of the flow manifested a significant curvature, which was attributed to second-normal stress difference. In some cases, the curved top surface was up to 30% higher than the margins of the flowing region. Cross-slope lateral circulations revealed details about the internal flow structure featuring a down-welling region in the middle and up-welling of material along the edges. Initial experiments and numerical simulations appeared to validate a theoretical model, based on a long-wave approximation, relating the height and velocity of the flow directly to the V-shape angle.

Three presentations discussed the effect of a viscous interstitial fluid on granular flow, spanning the range from a thin film of fluid coating the particles, to materials fully immersed in a fluid. Each presentation covered a different aspect of the effect of the interstitial fluid, and their results gave a glimpse of the richness and complexity of the mechanics of wet granular materials. Robert Davis studied the detailed micromechanics of colliding particles coated with a film of a wetting fluid. He observed the collision of two or more steel spheres coated with thin layers of viscous oil using a pendulum apparatus, the "Stokes Cradle." For oblique collisions of two spheres, the particles were observed to stick together at small impact velocities due to viscous forces, bounce apart at large impact velocities due to elastic deformation, and initially stick but then later separate at intermediate impact velocities and angles due to centrifugal forces on the rotating doublet. For head-on collisions of three spheres, all possible outcomes – agglomeration of all three spheres, one sphere separating from the remaining doublet, and all three spheres separating – were observed as the impact velocity and fluid thickness were varied.[39] He also discussed microphysical models to predict the collision dynamics and outcomes. Nico Gray first described experimental measurements of surface velocity near the advancing front of a water-saturated sand and gravel mixture in a debris-flow flume. Such flows often spontaneously develop large particle-rich levees that channelize the flow and enhance run-out. By tracking the motion of surface tracers and using a simple kinematic model, it was possible to infer the bulk three-dimensional motion as incoming material was sheared towards the front, over-run and shouldered to the side.[40] At the heart of the levee formation process is a subtle segregation-mobility feedback effect. The insights gained through analysis of simple models for segregation and the depth-averaged motion of granular avalanches were also discussed. Ashish Orpe presented the results of experiments on the dynamics of gravity-driven, dense granular materials fully immersed in an interstitial liquid. The experiments were performed in a slowly rotating cylinder half-filled with glass beads, with remaining space filled by the interstitial liquid. Different liquid viscosities are employed. Flow imaging and particle tracking was achieved by combinations of index-matching, laser-excited fluorescent dyes, and high-speed cinematography. The experimental velocity profiles are compared with the predictions of the models of Khakhar *et al.*[41] and Jop *et al.*[42] for dry granular flow.

In the first of two presentations dealing with particle migration and segregation in granular media, Devang Khakhar presented the results from DEM simulation of dense bidisperse granular flows on rough inclined planes, aimed at the study of rheology and segregation. A new model based on the visco-plastic model of Jop *et al.*[19] together with a Stokes-law drag model for gravitational sedimentation reportedly gives good agreement with simulations of equilibrium segregation in steady slow. Size segregation was modeled by means of effective-volume concepts. The second presentation by Kimberley Hill aimed at describing shear-induced size segregation in a gravity field. A model was proposed based on two mechanisms, one involving migration down a gradient in grain kinetic energy,[43] while the second mechanism involved size-sieving of the kind proposed by Gray and Thornton.[44] The model is reportedly validated by 3D DEM simulations of chute flow.

In most of the experiments on granular flows described so far, the particles are either spherical or nearly spherical (e.g, sand, gravel). The presentation of Tamás Börzsönyi was the only one to consider angular particles. He was concerned with the shear alignment of large ensembles of elongated objects, which is observed in contexts spanning a wide range of length scales: log jams on rivers, seeds, nano-rods and nematic liquid crystals. Shear alignment of macroscopic particles, observed through experiments and DEM simulations, was very similar to that seen in simple molecular systems, despite the completely different types of particle interactions. The preferred orientation of the grains forms a small angle with the streamlines, independent of shear rate across three decades, but decreases with increasing aspect ratio of the particles. The shear-induced alignment results in a considerable reduction of the effective friction of the granular material.[45]

The papers grouped under the "intermediate regime" covered a wide and varied spectrum of phenomena, in which various physical factors are at play: particle inertia, the effect of interstitial fluid (in saturated and partially saturated granular materials), size segregation, and particle shape. The micromechanical basis for rapid flows appears to be more sound, with the kinetic-theory approach providing a plausible starting point, but many open and challenging questions regarding correlations in dense dissipative systems remain. The DEM simulation technique continues to remain a powerful computational tool for the study of granular materials in all the regimes.

C. Turbulent fluid-particle flows

The topic of fluid turbulence and its interaction with particles is seldom found in most of the previous symposia on particle and particle-fluid flows. It was thought that a half-day session on the interaction between fluid turbulence on particle dynamics and the rheology of the suspension would be a fruitful exercise. The four papers presented in the session provide a flavor of the recent advances in modeling, simulation and experimental techniques of particle-laden turbulent flows.

Mike Reeks gave an overview of the subject, with a view towards modeling and simulating particle-laden turbulent fluid flows in order to describe the interaction of suspended particles with the large and small scale structures of the carrier-fluid turbulence. In this regard there are traditionally two ways of modeling such flows: an "Euler-Lagrange" stochastic approach[46] in which individual particles/droplets are tracked through a stochastically described turbulent flow field; and a strictly Eulerian two-fluid approach in which the dispersed (particle) phase and the carrier phase are each described by a set of continuum transport equations. The advantages and limitations of each approach were touched upon, as exemplified by recent works on the subject.[47] Reeks also noted the considerable improvement in experimental techniques, including PIV, X-ray, and radioactive imaging, to study dense suspensions of particles.

Following his overview, Reeks presented key results from his recent paper,[48] which was concerned with the segregation of particles in turbulent flows arising from particle interactions with the smaller dissipative scales. The paper describes the "full Lagrangian approach," which evaluates the compressibility of an elemental volume of particles as it moves through the carrier flow. Lagrangian statistics of this quantity can be accumulated in the same way as that of the particle velocity and position evaluated along an individual particle trajectory. One objective of the paper was to study the statistics of the segregation process in terms of the above compressibility, providing an extension of previous work.[49] The contribution from random uncorrelated motion and singularities were compatible with the formation of caustics.

The next two presentations demonstrated the use of the Euler-Lagrange approach, mentioned above. Toshitsugu Tanaka used the approach to numerically simulate the motion of a large particle in a dense (Geldart A) gas-solid flow using a DEM-CFD coupling method.[50] In this method, the spatial resolution of the fluid motion is taken to be larger than the particle size but smaller than the mesoscale structures developing in the flows (bubbles in the case of fluidized beds). Tanaka used this method to investigate the motion of a large sphere (generated by a large number of dense small virtual particles) in a bubbling fluidized bed, and compared the results with experiment. Partha Goswami used the same method, but determined the particle motion in greater detail. He used a fluctuating force model for the effects of turbulent velocity fluctuations on the particles in the limit of high Stokes numbers, where the particle relaxation time is much greater than the turbulent integral time scales. Treating the driving force on the particles as an anisotropic white noise in a Langevin model, he showed predictions of the velocity distribution in the presence of a downward gravitational force. Good agreement was found with the results obtained from the DNS generated velocity field. Comparison was also made with experimental results obtained from PIV and particle tracking velocimetry measurements. Agreement with the Langevin stochastic model was obtained only if polydispersity in particle size was incorporated.

The final paper in the session, presented by Shantanu Roy, was concerned with the identification of flow structures in experiments on multiphase flow, using radioactive particle tracking (RPT).[51] The focus of the presentation was RPT data treatment based on the theory of non-linear dynamical systems (NLDS). The time series of the tracer location serves as a signature of the flow pattern and, hence, the flow regimes that exist in the multiphase system. NLDS parameters characterizing flow (such as Kolmogorov entropy) were evaluated and their physical interpretation was discussed in terms of the flow pattern and underlying forces. As examples, a gas-liquid system (air-water bubble column) and a gas-solid system (unary and binary fluidized bed) were considered.

Significant advances have been made in the modelling and computation of gas-solid flows in the recent years, with modules available in standard computational fluid dynamics (CFD) packages such as CFX and FLUENT. However, some fundamental issues remain to be resolved. This session brought out some of the main advances, such as the sophistication of the two-phase flow models, and the methods for their solution. It also highlighted some of the important open problems, such as the effect of fluid turbulence on the particles, the nature of inter-particle interactions in a turbulent flow and the associated statistics, and the effect of polydispersity in particle size.

IV. LIVING SUSPENSIONS

In recent years, the study of particle-fluid flow has been extended to the biological realm, such as in suspensions of swimming microorganisms and the flow of blood. The early fluid-dynamical investigations in this field involved the study of individual swimmers or the shape and motion of a single cell, but this has evolved into the study of a collection of swimmers or cells. This is an important new direction in the field of suspension mechanics, which holds significant promise in the understanding of important problems in biology. The fifth and final day of the symposium focused on this area of living suspensions. The session included theoretical, computational, and experimental studies on a number of topics related to living suspensions, including mechanisms for propulsion, the effects of interactions between swimmers, artificial swimmers, and the effective rheology of active suspensions.

The session opened with an overview of the field by Sriram Ramaswamy, which started with a review of mechanisms for swimming in the absence of inertia, including G. I. Taylor's swimming sheet, Purcell's three-link swimmer, and helical swimming. He noted that the minimal swimmer model for the understanding of interactions between motile microorganisms or artificial swimmers is a force dipole in general. At a coarse-grained level, these dipoles drive active stresses that can lead to instabilities[52] and a non-Newtonian rheology. He also argued that living suspensions share many similarities with other physical and biophysical systems, including certain granular materials, fish shoals, and the cytoskeleton of the cell. The majority of the presentations that followed were devoted to continuum theories and simulations for active suspensions, but a few presentations discussed new experimental measurements.

During the course of the day, four papers were presented on continuum theories for active suspensions. In all the theories, the fluid velocity field interacts with the orientation of the active swimmers, which in turn exerts an active stress on the fluid. The theories in general assume an active swimmer to orient in response to the ambient fluid flow in the same manner as a passive particle, and an evolution equation for the continuum orientation field is solved in conjunction with the Stokes equations for the suspension. Sriram Ramaswamy followed his overview to describe his recent work, wherein he has extended the equations of nematic hydrodynamics to include the active stress. First, he analyzed the spreading of a liquid drop of an active suspension on a rigid substrate for certain assumed orientation fields, to understand the effects of active stresses (which are related to the local orientation of the particles) on the rate of spreading of the drop.[53] Using a lubrication approximation, he showed that, depending on the nature of the orientation field, different spreading rates can arise. He then described a theoretical study of the chemotactic motion of catalytic colloids in a chemical gradient, where he showed that the ability of such colloids to respond to the gradient depends on the surface distribution of the reaction sites on the particle surface. Michael Graham followed with a presentation on the statistics in large-scale suspensions of self-propelled particles. Using a simple dilute theory, he derived scalings for the dependence of swimmer and passive tracer diffusivities on swimmer volume fraction, and showed that fluctuations in these systems can be interpreted in terms of an effective suspension temperature T_{eff}, obtained by comparing the magnitude of active stresses to that of thermal fluctuating stresses.[54] He finds that $T_{\mathrm{eff}} \approx 10^3\, T$ in many situations (where T is the actual temperature), which is consistent with previous experimental measurements.[55] He also described recent work on the synchronization of rotating flagella (such as those on most bacteria), modeled as a string of rigid slender bodies. In the case of two flagella, he showed that hydrodynamic interactions can lead to synchronization and bundling, with the existence of both loose and tight bundled states.

Later, Ganesh Subramanian discussed a general theoretical framework for the modeling of the dynamics in living suspensions.[56] First, a model based on a singlet probability function was presented, in which the effects of mean-field hydrodynamic interactions as well as bacterial tumbling were included. Based on this model, he showed that uniform isotropic suspensions of pushers become unstable to long-wave fluctuations above a critical concentration, causing them to align locally. This instability may also be interpreted as a result of the decrease in effective viscosity due to self-propulsion. Subramanian then extended this theory to calculate pair probabilities in the stable regime, using slender-body theory to regularize near-field interactions, and a multiple-scale analysis for far-field interactions. In the final theoretical paper, David Saintillan presented a kinetic model and continuum simulations of the dynamics of a thin film of active suspension. The model extends his previous work[57] on bulk suspensions and captures mean-field hydrodynamic interactions between bacteria, the effect of confinement, and chemotactic interactions with an oxygen concentration field that diffuses from the boundaries, is transported by the flow, and consumed by the swimmers. Three-dimensional simulations show the existence of a transition from steady dynamics in thin films to unsteady chaotic dynamics with enhanced oxygen transport in thicker films, in agreement with previous experiments.[58] He also described a more recent extension of this model that captures some of the effects of near-field steric interactions in the concentrated regime, by means of an additional interaction energy causing local particle alignment.

While the existing continuum theories are restricted to dilute suspensions of weakly-interacting swimmers, numerical simulations offer the possibility of accounting for the hydrodynamic interactions between the swimmers, and also computing the statistics of swimmer velocities and orientations. Philippe Peyla's paper discussed numerical simulations, focusing on the rheology of suspensions of active rotors, or spherical particles on which a constant external torque is applied. While active rotors may not be found in living suspensions, they can be obtained artificially in the lab, for instance by electro-rotation. Because of the torque each particle transmits to the fluid, the effective shear viscosity of a suspension of rotors is modified and can be either reduced or enhanced depending on the relative direction of the torque and of the flow vorticity. A theory for the intrinsic viscosity for dilute suspensions is already available,[59] and Peyla described an extension to higher concentrations using numerical simulations based on the "fluid particle dynamics" method.[60] He showed that the dependence of the effective viscosity on rotor angular velocity collapsed to a single master curve for all the volume fractions studied.

Two papers presented the results of experimental investigations on living suspensions. Salima Rafaï described experiments on the dynamics of the alga *Chlamydomonas reinhardtii*, which is used in many investigations of living suspensions. Using high-speed imaging, she showed that the trajectories of swimming *Chlamydomonas* are well modeled as a correlated random walk,[61] with three time scales corresponding to the cilia beating motions, a ballistic regime, and a diffusive regime. She also presented results on the effective rheology of algal suspensions, which exhibit shear-thinning and enhanced shear viscosities as a result of activity. Finally, she concluded by describing the ability to focus *Chlamydomonas* near the centerline of a pressure-driven Poiseuille flow when light is shone from upstream. An analogy was drawn between this effect, a result of the phototactic response of the algae, and the gravitactic focusing of bottom-heavy swimmers.[62] Ranganathan Prabhakar presented experiments aimed at measuring the extensional viscosity of active suspensions to validate recent theoretical predictions in the dilute regime.[63] He underlined the difficulty of such measurements using conventional extensional rheometers because of the low viscosity of the samples and of the low extensional rates at which effects are seen. He argued that a suitable tool for such measurements is a microfluidic rheometer based on liquid bridges actuated by surface acoustic waves.[64] Measurements show that suspensions of bacteria have a decreased viscosity due to swimming, as demonstrated by a shorter time for breakup of the liquid bridges, whereas suspensions of algae show an enhanced viscosity.

Analyses of the detailed mechanics of propulsion of individual swimmers were given in two presentations. Living suspensions have inspired engineers to design artificial swimmers in the lab,[65] often by means of a chemical reaction on the particle surface of a solute in the suspending liquid. In his presentation, John Brady described a model for the propulsion of a catalytic nano-motor based on self-diffusiophoresis.[66] His analysis considered the motion of a large colloidal sphere suspended in a bath of smaller colloidal particles, representing the solute – the continuum limit can be recovered by letting the size of the smaller particles vanish. Using both theory and Brownian dynamics simulations, he showed that a nonuniform reactivity on the surface of the motor leads to self-propulsion by a mechanism analogous to an osmotic pressure force. Ashok Sangani described his work on the beating mechanisms of the cilium, the filament that propels eukaryotic cells and also regulates the motion of fluids through various organs in mammals. Several mechanisms have been proposed in the past, but no consensus has been reached so far on the relation between the forces generated by internal motors and the cilium kinematics. Among the most popular mechanisms are the load-dependent detachment model[67] and the geometric clutch model,[68] which Ashok Sangani proceeded to test by examining existing experimental data on beating cilia within the framework of a slender-body model, and comparing the results of his model to these existing theories. Preliminary results appear to support the geometric clutch model, though additional work is needed.

It is clear from the day's proceedings that the sophistication of the theories and simulation techniques for active suspensions has grown considerably over the past decade. However, most of the presentations considered active suspensions wherein interactions between swimmers is only through the viscous forces exerted by the fluid, though preliminary efforts to include near-field steric interactions are being attempted. The experiments too try to replicate systems where such purely physical interactions are dominant. However, real problems in biology involve a variety of interactions between the microorganisms and their surrounding environment, at a variety of time scales. The ability of the simplest of microorganisms, such as bacteria, to communicate and behave in a coordinated fashion (quorum sensing) is an example. It is hoped that the models and simulations of the future will build in these phenomena, so that such problems may be addressed.

[1] The scientific committee appointed by the IUTAM was: John F. Brady (California Institute of Technology, USA), Joe D. Goddard (University of California at San Diego, USA), Elisabeth Guazzelli (Aix-Marseille Universite, France), Hisao Hayakawa (Kyoto University, Japan), Devang Khakhar (Indian Institute of Technology Bombay, India), L. Gary Leal (University of California at Santa Barbara, USA), Prabhu R. Nott (Chair, Indian Institute of Science, India), and Sankaran Sundaresan (Princeton University, USA).

[2] See supplementary material at http://dx.doi.org/10.1063/1.4812639 for the complete programme and book of abstracts from the symposium.

[3] E. Guazzelli and L. Oger, *Mobile Particulate Systems, NATO ASI Series E: Applied Sciences* (Kluwer Academic Publishers, Dordrecht, 1995).

[4] These include the *IUTAM Symposium on Hydrodynamic Diffusion of Suspended Particles* held in 1995 at Estes Park, USA,[69] the *IUTAM Symposium on Interactions for Dispersed Systems in Newtonian and Viscoelastic Fluids* held in 2006 at Guanajuato, Mexico,[70] the *IUTAM Symposium on Recent Advances in Multiphase Flow* held in 2008 at Istanbul, Turkey,[71] and the *IUTAM-ISIMM Symposium on Mathematical Modeling and Physical Instances of Granular Flows* held in 2009 at Reggio Calabria, Italy.[72]

[5] R. E. Caflisch and J. H. Luke, "Variance in the sedimentation speed of a suspension," Phys. Fluids **28**, 759 (1985).

[6] H. Nicolai and E. Guazzelli, "Effect of the vessel size on the hydrodynamic diffusion of sedimenting spheres," Phys. Fluids **7**, 3 (1995).

[7] J. H. Luke, "Decay of velocity fluctuations in a stably stratified suspension," Phys. Fluids **12**, 1619 (2000).

[8] D. C. Gómez, L. Bergougnoux, J. Hinch, and E. Guazzelli, "On stratification control of the velocity fluctuations in sedimentation," Phys. Fluids **19**, 098102 (2007).

[9] J. Nitsche and G. Batchelor, "Break-up of a falling drop containing dispersed particles," J. Fluid Mech. **340**, 161–175 (1997).

[10] Y. Oyama, Bull. Inst. Phys. Chem. Res. Jpn. Report 5 **18**, 600 (1939).

[11] S. Das Gupta, D. Khakhar, and S. Bhatia, "Axial segregation of particles in a horizontal rotating cylinder," Chem. Eng. Sci. **46**, 1513–1517 (1991).

[12] M. Tirumkudulu, A. Tripathi, and A. Acrivos, "Particle segregation in monodisperse sheared suspensions," Phys. Fluids **11**, 507 (1999).

[13] G. Batchelor and J. Green, "The determination of the bulk stress in a suspension of spherical particles to order c^2," J. Fluid Mech. **56**, 401–427 (1972).

[14] H. J. Wilson and R. H. Davis, "The viscosity of a dilute suspension of rough spheres," J. Fluid Mech. **421**, 339–367 (2000).

[15] F. Boyer, É. Guazzelli, and O. Pouliquen, "Unifying suspension and granular rheology," Phys. Rev. Lett. **107**, 188301 (2011).

[16] É. Couturier, F. Boyer, O. Pouliquen, and É. Guazzelli, "Suspensions in a tilted trough: second normal stress difference," J. Fluid Mech. **686**, 26 (2011).

[17] S. Savage, "The mechanics of rapid granular flows," Adv. Appl. Mech. **24**, 289–366 (1984).

[18] G. D. R. Midi, "On dense granular flows," Eur. Phys. J. E **14**, 341–365 (2004).

[19] P. Jop, Y. Forterre, and O. Pouliquen, "A constitutive law for dense granular flows," Nature (London) **441**, 727–730 (2006).

[20] O. Reynolds, "On the dilatancy of media composed of rigid particles in contact. With experimental illustrations," Philos. Mag. **20**, 469–481 (1885).

[21] A. N. Schofield and C. P. Wroth, *Critical State Soil Mechanics* (McGraw-Hill, London, 1968).

[22] R. Jackson, "Some mathematical and physical aspects of continuum models for the motion of the granular materials," in *Theory of Dispersed Multiphase Flow*, edited by R. E. Meyer (Academic Press, New York, 1983), pp. 291–337.

[23] K. K. Rao and P. R. Nott, *An Introduction to Granular Flow* (Cambridge University Press, New York, 2008).

[24] D. G. B. Edelen, "On the existence of symmetry relations and dissipation potentials," Arch. Ration. Mech. Anal. **51**, 218–227 (1973).

[25] P. Dantu, "Contribution à l'étude mécanique et géometrique des milieux pulveérulents," in *Proceedings of the 4th International Conference on Soil Mechanics and Foundations Engineering* (Butterworths, London, 1957), pp. 144–148.

[26] V. Mehandia, K. J. Gutam, and P. R. Nott, "Anomalous stress profile in a sheared granular column," Phys. Rev. Lett. **109**, 128002 (2012).

[27] H. B. Mühlhaus and I. Vardoulakis, "The thickness of shear bands in granular materials," Geotechnique **37**, 271–283 (1987).

[28] I. Vardoulakis and E. Aifantis, "A gradient flow theory of plasticity for granular materials," Acta Mech. **87**, 197–217 (1991).

[29] L. S. Mohan, K. K. Rao, and P. R. Nott, "A frictional Cosserat model for the slow shearing of granular materials," J. Fluid Mech. **457**, 377–409 (2002).

[30] T. Liu and L. Shen, "Fluid flow and optical flow," J. Fluid Mech. **614**, 253–291 (2008).

[31] I. Sharma, J. T. Jenkins, and J. A. Burns, "Dynamical passage to approximate equilibrium shapes for spinning, gravitating rubble asteroids," Icarus **200**, 304–322 (2009).

[32] W. M. Visscher and M. Bolsterli, "Random packing of equal and unequal spheres in two and three dimensions," Nature (London) **239**, 504–507 (1972).

[33] T. Schwager, D. E. Wolf, and T. Pöschel, "Fractal substructure of a nanopowder," Phys. Rev. Lett. **100**, 218002 (2008).

[34] S. Chialvo, J. Sun, and S. Sundaresan, "Bridging the rheology of granular flows in three regimes," Phys. Rev. E **85**, 021305 (2012).
[35] T. Weinhart, A. R. Thornton, S. Luding, and O. Bokhove, "From discrete particles to continuum fields near a boundary," Granular Matter **14**, 289–294 (2012).
[36] J. Jenkins and M. Richman, "Kinetic theory for plane flows of a dense gas of identical, rough, inelastic, circular disks," Phys. Fluids **28**, 3485 (1985).
[37] K. Saitoh and H. Hayakawa, "Weakly nonlinear analysis of two dimensional sheared granular flow," Granular Matter **13**, 697–711 (2011).
[38] F. Pacheco-Vázquez, G. Caballero-Robledo, J. Solano-Altamirano, E. Altshuler, A. Batista-Leyva, and J. Ruiz-Suárez, "Infinite penetration of a projectile into a granular medium," Phys. Rev. Lett. **106**, 218001 (2011).
[39] C. Donahue, C. Hrenya, and R. Davis, "Stokes' cradle: Newton's cradle with liquid coating," Phys. Rev. Lett. **105**, 034501 (2010).
[40] C. Johnson, B. Kokelaar, R. Iverson, M. Logan, R. LaHusen, and J. Gray, "Grain-size segregation and levee formation in geophysical mass flows," J. Geophysical Res. **117**, F01032, doi:10.1029/2011JF002185 (2012).
[41] D. Khakhar, A. V. Orpe, and J. Ottino, "Surface granular flows: two related examples," Adv. Complex Syst. **4**, 407–417 (2001).
[42] P. Jop, Y. Forterre, O. Pouliquen *et al.*, "Crucial role of sidewalls in granular surface flows: Consequences for the rheology," J. Fluid Mech. **541**, 167 (2005).
[43] Y. Fan and K. Hill, "Phase transitions in shear-induced segregation of granular materials," Phys. Rev. Lett. **106**, 218301 (2011).
[44] J. Gray and A. Thornton, "A theory for particle size segregation in shallow granular free-surface flows," Proc. R. Soc. London **461**, 1447–1473 (2005).
[45] T. Börzsönyi, B. Szabó, G. Törös, S. Wegner, J. Török, E. Somfai, T. Bien, and R. Stannarius, "Orientational order and alignment of elongated particles induced by shear," Phys. Rev. Lett. **108**, 228302 (2012).
[46] M. Sommerfeld, "Validation of a stochastic Lagrangian modelling approach for inter-particle collisions in homogeneous isotropic turbulence," Int. J. Multiphase Flow **27**, 1829–1858 (2001).
[47] O. Simonin, E. Deutsch, and J. P. Minier, *Eulerian Prediction of the Fluid/Particle Correlated Motion in Turbulent Two-phase Flows* (Springer, 1993).
[48] E. Meneguz and M. W. Reeks, "Statistical properties of particle segregation in homogeneous isotropic turbulence," J. Fluid Mech. **686**, 338 (2011).
[49] R. H. A. Ijzermans, E. Meneguz, and M. W. Reeks, "Segregation of particles in incompressible random flows: singularities, intermittency and random uncorrelated motion," J. Fluid Mech. **653**, 99 (2010).
[50] Y. Tsuji, T. Kawaguchi, and T. Tanaka, "Discrete particle simulation of two-dimensional fluidized bed," Powder Technol. **77**, 79–87 (1993).
[51] S. Roy, A. Kemoun, M. H. Al-Dahhan, and M. P. Dudukovic, "Experimental investigation of the hydrodynamics in a liquid–solid riser," AIChE J. **51**, 802–835 (2005).
[52] A. R. Simha and S. Ramaswamy, "Hydrodynamic fluctuations and instabilities in ordered suspensions of self-propelled particles," Phys. Rev. Lett. **89**, 058101-1–058101-4 (2002).
[53] J.-F. Joanny and S. Ramaswamy, "A drop of active matter," J. Fluid Mech. **705**, 46–57 (2012).
[54] P. T. Underhill and M. D. Graham, "Correlations and fluctuations of stress and velocity in suspensions of swimming microorganisms," Phys. Fluids **23**, 121902 (2011).
[55] X.-L. Wu and A. Libchaber, "Particle diffusion in a quasi-two-dimensional bacterial bath," Phys. Rev. Lett. **84**, 3017–3020 (2000).
[56] G. Subramanian and D. L. Koch, "Critical bacterial concentration for the onset of collective swimming," J. Fluid Mech. **632**, 359 (2009).
[57] D. Saintillan and M. J. Shelley, "Instabilities, pattern formation, and mixing in active suspensions," Phys. Fluids **20**, 123304 (2008).
[58] A. Sokolov, R. E. Goldstein, F. I. Feldchtein, and I. S. Aranson, "Enhanced mixing and spatial instability in concentrated bacterial suspensions," Phys. Rev. E **80**, 031903 (2009).
[59] H. Brenner, "Rheology of a dilute suspension of dipolar spherical particles in an external field," J. Colloid Interface Sci. **32**, 141–158 (1970).
[60] L. Jibuti, S. Rafaï, and P. Peyla, "Suspensions with a tunable effective viscosity: A numerical study," J. Fluid Mech. **693**, 345–366 (2012).
[61] M. Garcia, S. Berti, P. Peyla, and S. Rafaï, "Random walk of a swimmer in a low-Reynolds-number medium," Phys. Rev. E **83**, 035301 (2011).
[62] J. O. Kessler, "Hydrodynamic focusing of motile algal cells," Nature (London) **313**, 218–220 (1985).
[63] D. Saintillan, "Extensional rheology of active suspensions," Phys. Rev. E **81**, 056307 (2010).
[64] P. Bhattacharjee, A. McDonnell, R. Prabhakar, L. Yeo, and J. Friend, "Extensional flow of low-viscosity fluids in capillary bridges formed by pulsed surface acoustic wave jetting," New J. Phys. **13**, 023005 (2011).
[65] S. J. Ebbens and J. R. Howse, "In pursuit of propulsion at the nanoscale," Soft Matter **6**, 726–738 (2010).
[66] J. F. Brady, "Particle motion driven by solute gradients with application to autonomous motion: Continuum and colloidal perspectives," J. Fluid Mech. **667**, 216–259 (2011).
[67] S. Camalet and F. Julicher, "Generic aspects of axonemal beating," New J. Phys. **2**, 24 (2000).
[68] C. B. Lindemann, "A geometric clutch hypothesis to explain oscillations of the axoneme of cilia and flagella," J. Theor. Biol **168**, 175–189 (1994).
[69] R. Davis, "Hydrodynamic diffusion of suspended particles: A symposium," J. Fluid Mech. **310**, 325–335 (1996).
[70] M. Denn, E. Meiburg, J. Morris, E. Shaqfeh, and T. Squires, "Report of the symposium on interactions for dispersed systems in Newtonian and viscoelastic fluids," Phys. Fluids **18**, 121501 (2006).

[71] A. Acrivos and C. Delale, "Report on the IUTAM Symposium on Recent Advances in Multiphase Flows: Numerical and Experimental (11–14 June 2007, Istanbul, Turkey)," Phys. Fluids **20**, 040501 (2008).
[72] *IUTAM-ISIMM Symposium on Mathematical Modeling and Physical Instances of Granular Flows, Reggio Calabria, Italy, 14–18 September 2009*, AIP Conference Proceedings Vol. 1227, edited by J. Goddard, J. Jenkins, and P. Giovine (American Institute of Physics, 2010).